$$Q_n > (2p_1)^{(p_2-1)\cdots(p_s-1)} \div 2^{2^{s-2}} > p_1,$$

since at least $s-2$ of the primes $p_2, \ldots\ldots, p_s$ exceed 2, so that the exponent is $\overline{\gtrless} 2^{s-2}$. For $s=2$, we have $p_1 \overline{\gtrless} 3$, $Q_n > \frac{1}{2}y^{(p_1-1)(p_2-1)} \overline{\gtrless} p_1$ unless $p_1=3$, $y^{p_2-1}=2$, whence $p_2=2$, $y=2$, $n=6$, $x=2$.

Corollary. *If x is a positive integer >1, x^n-1 has a prime factor not dividing x^m-1 $(m<n)$, except in the cases $n=2$, $x=2^k-1$; $n=6$, $x=2$.*

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Problems 219, 220 were also solved by L. E. Newcomb. No. 222 was also solved by A. H. Holmes.

223. Proposed by **THEODORE L. DE LAND**, Office of the Secretary of the Treasury, Washington, D.C.

An officer in the Treasury Department assigned three clerks to count a lot of silver dollars and when finished noted that there was an apparent difference in their efficiency; and, to determine the fact, gave to each a similar lot of the same amount to count, the only record made at the time being that A to count his lot alone, took three weeks longer, B took two weeks longer, and C took one week longer than it took for all working together to count the first lot. The best counter, on the record made, was given an efficiency mark of 93 on the scale of 100. What efficiency mark should, on the record, be given to each of the other two counters?

Solution by the PROPOSER.

Let x=the time for A, B, and C working together to finish one lot.
Then $x+3$=the time for A to finish one lot working alone;
$x+2$=the time for B to finish one lot working alone; and
$x+1$=the time for C to finish one lot working alone.

Then $\dfrac{1}{x}$=what A, B, and C can do in one week working together;

$\dfrac{1}{x+3}$=what A can do in one week working alone;

$\dfrac{1}{x+2}$=what B can do in one week working alone; and

$\dfrac{1}{x+1}$=what C can do in one week working alone.

Equating like terms we have,

$$\frac{1}{x}=\frac{1}{x+3}+\frac{1}{x+2}+\frac{1}{x+1} \quad \ldots\ldots\ldots (1).$$

Reducing, we have,

$$x^3+3x^2-3=0\ \ldots\ldots\ (2).$$

By Horner's method, we have from equation (2), $x=0.879385+$. Therefore

$$\frac{1}{x+3}=\frac{1}{3.839385};\quad \frac{1}{x+2}=\frac{1}{2.879385};\ \text{and}\ \frac{1}{x+1}=\frac{1}{1.879385}.$$

It is evident that C is the best clerk and was given the 93% on the efficiency record. The records should be inversely proportional to the time expended for equivalent work. In order to compare C and B, and C and A, we have

$$x+2 : x+1=93\% : B\text{'s mark};$$
$$x+3 : x+1=93\% : A\text{'s mark};$$

and therefore,

$$2.879385 : 1.879385=93\% : 60.70\%=B\text{'s mark; and}$$
$$3.879385 : 1.879385=93\% : 45.05\%=A\text{'s mark}.$$

Thus, if C were given on the efficiency record 93%, A should be given 45.05%, and B should be given 60.70%.

Also solved by G. B. M. Zerr, S. A. Corey, G. W. Greenwood, F. D. Whitlock, R. D. Carmichael, A. H. Holmes, and J. Scheffer.

224. Proposed by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

Show that, if none of the quantities x, y, z is zero, the result of eliminating them from

$$(x+y)(x+z)=bcyz\ \ldots\ldots\ (1),$$
$$(y+z)(y+x)=cazx\ \ldots\ldots\ (2),$$
$$(z+x)(z+y)=abxy\ \ldots\ldots\ (3),$$

is $\begin{vmatrix} \pm a, & 1, & 1 \\ 1, & \pm b, & 1 \\ 1, & 1, & \pm c \end{vmatrix}=0.$ [Oxford, 1896.]

Solution by C. H. MILLER, West Point, N. Y., and the PROPOSER.

By multiplying the second equation by the third, dividing by the first, and transposing, we obtain

$$\pm ax+g+z=0.$$

From this, and two similar equations, we get the required elimininant.

Also solved by J. B. Faught, G. B. M. Zerr, R. D. Carmichael, J. Scheffer, and J. O. Mahoney.

225. Proposed by H. M. ARMSTRONG, Cooch's Bridge, Delaware.

If $\alpha=ax+cy+bz\ \ldots\ldots\ (1)$, $\beta=cx+by+az\ \ldots\ldots\ (2)$, $\gamma=bx+ay+cz\ \ldots\ldots\ (3)$, show that $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=(a^3+b^2+c^3-3abc)(x^3+y^3+z^3-3xyz)$.

Solution by the PROPOSER.

The required result follows directly from the equality,